

The Cambridge Machine for Measuring Celestial Photographs.
By Arthur R. Hinks, M.A.

As a preliminary to the preparation, some three years ago, of plans for a new measuring machine for the Cambridge Observatory, the Director authorised me to prepare an account of the methods which had at that time been proposed and of the machines which had been made for measuring celestial photographs. Some notes from this account may serve as an introduction to the description of our new machine.

1. *Types of Modern Measuring Machines.*

Modern machines for measuring rectangular coordinates may be divided broadly into two types :

Type A, in which positions upon the photographic plate are referred to a standard scale on the measuring machine.

Type B, in which a *réseau* impressed upon the plate takes the place of the standard scale, and it is necessary only to measure the place of the star image within a *réseau* square.

There are two leading forms of machines of type A.

A 1, the Repsold machine described by Bakhuyzen (*Bull. Com. Perm.* I. 164), in which the microscope tilts on its chariot, and is pointed successively on the plate and on the standard scale.

A 2, various machines, like the Zeiss "Comparator," in which there are two microscopes rigidly connected together, one to view the plate, the other to view the standard scale.

It is required in both these forms that the main slides shall be geometrically perfect, or that their errors shall be investigated and applied ; that the temperature corrections of the scale and plate shall be determined or in some way eliminated ; and it is further required in form A 1 that the tilting motion of the microscope shall be perfect. These are all serious impediments. Moreover, it is almost universally admitted that to control the distortion of the gelatine film it is necessary to impress upon the plate a *réseau*. To refer the *réseau* lines as well as the star images to a standard scale, so that the distortions of the film are actually determined over the whole plate, is cumbrous. Historically this is the original use of the *réseau*. But it was quickly realised that its true use is to treat it as the standard scale. This proceeding straightway eliminates all necessity for considering film distortion except within the one *réseau* square in which the star image to be measured lies. Within this square the distortion must be considered uniform, showing itself as error of run (after correcting for errors of division of the *réseau*), and

eliminated in the usual way when the star image is referred to the four sides of the square which enclose it.

For the measurement of plate impressed with a *réseau* whose errors are known or which may be neglected, it is sufficient to use a machine of type B. Of such machines there were three years ago two principal forms :—

B 1, the machines used at Paris, Potsdam, and elsewhere, consisting of a microscope with field of view sufficiently large to take in a R-square (5 mm.) and a filar micrometer in the eyepiece to measure the distance of the star image from the sides of the square.

B 2, the machines used at Greenwich and Oxford, in which these distances were measured by estimation on a finely divided glass scale in the eyepiece.

Both these forms possess structural advantages over machines of type A. The principal slides which carry the plate under the microscope need not be very accurate. They can produce no errors in measurement unless they are so bad as to tilt the plate sensibly.

On the other hand, machines of type B 1 have one grave disadvantage, which is shared in some degree by all machines with micrometer screws. They involve a great deal of turning of the screws, which wastes time and wears out the screws. In the Potsdam machine, for example, the screw makes about 1,500 revolutions per hour. A complete measure requires in general about twenty turns, ten forward and ten back.

Machines of type B 2 have the advantage that an invariable glass scale is used instead of a screw ; but the last place in the readings is only an estimate of the proportion in which the scale interval is divided by the image of a R-line. The accuracy with which this estimate can be made is shown to be only just enough for the purposes of the Astrographic Catalogue, when something must be sacrificed to speed ; but it is not enough for work of any greater refinement.

2. *The Principle adopted for the Cambridge Machine.*

Since the *réseau* is always used at Cambridge the machine was to be of type B. I proposed to try a combination of the essential features of the two machines of this type described above, and to use a divided glass scale moved in the microscope by a micrometer screw. In this method the chief part of the distance to be measured is read off on the scale ; the fractional part of the scale space is not estimated, but measured by the screw, which carries the results to one more place of decimals. Since it is easier to place a star disc or a R-line symmetrically between two divisions than bisected by one, the spaces of the scale should be numbered instead of the divisions. The screw turns in a fixed nut, and its point bears against the frame

carrying the scale. The head of the screw should be divided so that readings of the head increase as it is turned to make the scale readings on the object diminish; the sum of the readings upon a fixed object of scale and screw, reduced to the same units, then remains constant.

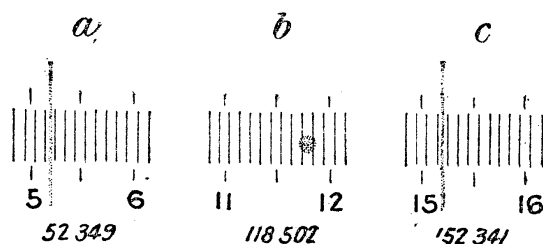


Fig. 1.

The method of making a measure on this plan is illustrated by the annexed diagram (fig. 1), which is drawn to represent the proportions which were adopted.

The R-interval is 5 mm.; 100 spaces of the scale = 1 R-interval; and the pitch of the micrometer screw is $0^{\text{mm}}.5 = 10$ scale spaces.

The R-square is brought into the field, and it is required to measure the distance of the star S from the line A.

The scale is moved by the screw until the nearest space is bisected by line A.

Scale reading 52, screw .349 say (fig. 1a).

The scale is then moved so that the star is placed symmetrically on an adjacent space.

Scale 118, screw .502 (fig. 1b)

And finally the line B is made to bisect a space.

Scale 152, screw .341 (fig. 1c)

The zero is of course perfectly arbitrary.

Now these numbers expressed as decimals of a R-interval are :

$$\left. \begin{array}{l} \text{A} \quad 0.52 \quad 0.0349 \\ \text{S} \quad 1.18 \quad .0502 \\ \text{B} \quad 1.52 \quad .0341 \end{array} \right\} \quad \dots \quad \dots \quad \dots \quad (1)$$

And adding corresponding readings of screw to scale we have :

$$\left. \begin{array}{l} \text{A} \quad 0.5549 \\ \text{S} \quad 1.2302 \\ \text{B} \quad 1.5541 \end{array} \right\} \quad \dots \quad \dots \quad \dots \quad (2)$$

It follows that the run is $\cdot 0008$, correction positive, and the corrected distance of star from A is

$$\cdot 6753 + \cdot 0005 = \cdot 6758.$$

And it should be noted that the result is the same, whichever spaces are bisected, so long as the ratio of screw to scale is correct. We might have used different spaces and obtained the readings—

$$\begin{array}{r} 50 \quad 549 \\ 119 \quad 402 \\ 154 \quad 141 \end{array}$$

and these would reduce as before to the numbers of (2).

This arrangement seemed to possess the following advantages. The whole length of screw used in making the measure need never be more than one or two tenths of a revolution ; there is therefore no time lost in running the screw backwards and forwards, and there is very little wear. It is scarcely possible that progressive error of the screw can come in ; and since repeated settings should be made on different spaces, and consequently with different parts of the screw, periodic error would always tend to go out.

Finally, if it is desired to make rough preliminary measurements, the screws may be left untouched, and measures made by estimation on the scale, as in the Greenwich and Oxford machines.

3. *Details which are desirable.*

In order to be able to measure in both coordinates at once, it is desirable that the scale in the microscope should consist of two scales crossing at right angles and moved by two screws at right angles. Further, to obtain the full power of using the scale for estimation in the Oxford method, the eyepiece must command a field two R-intervals at least in diameter ; and this is best accomplished by putting it on a slipping piece.

The microscope objective which projects an image of the plate on the micrometer scale is best placed to give a magnification unity. This arrangement possesses the convenient property that the image of the R-square can be made to fit the scale, and the error of run reduced to zero by racking the objective in or out with respect to its tube. No further adjustment is required. The plate remains in focus on the scale for a considerable range of motion of the objective, since for magnification unity the distance between conjugate foci of the lens is a minimum. This arrangement of the objective seems to be in common use, but its advantage has sometimes been lost because the objective has not been given a power of adjustment independent of the microscope tube.

A reversing prism in the microscope, by means of which the field of view can be rotated, is very useful in avoiding personality in measurement. In my own case it is a necessity, because the astigmatism of my eyes makes it impossible for me to see vertical lines as well as horizontal. The power of readily turning the field until the divisions of the scale I am using are horizontal is then a great convenience.

It is a well-known inconvenience of working with a micrometer that in making the settings the eye has to be close down to the micrometer heads, and must be withdrawn to read the heads. A minor inconvenience is that the nose has frequently to be pressed flat against the micrometer box. These difficulties can be avoided by using a compound microscope of low power instead of an eyepiece to view the micrometer scale. The divided heads are then further away from the eye, and in a position more convenient to handle. With the present machine it is easy to read the head with one eye without removing the other from the eyepiece.

It is an immense convenience if all adjustable parts are furnished with divided scales.

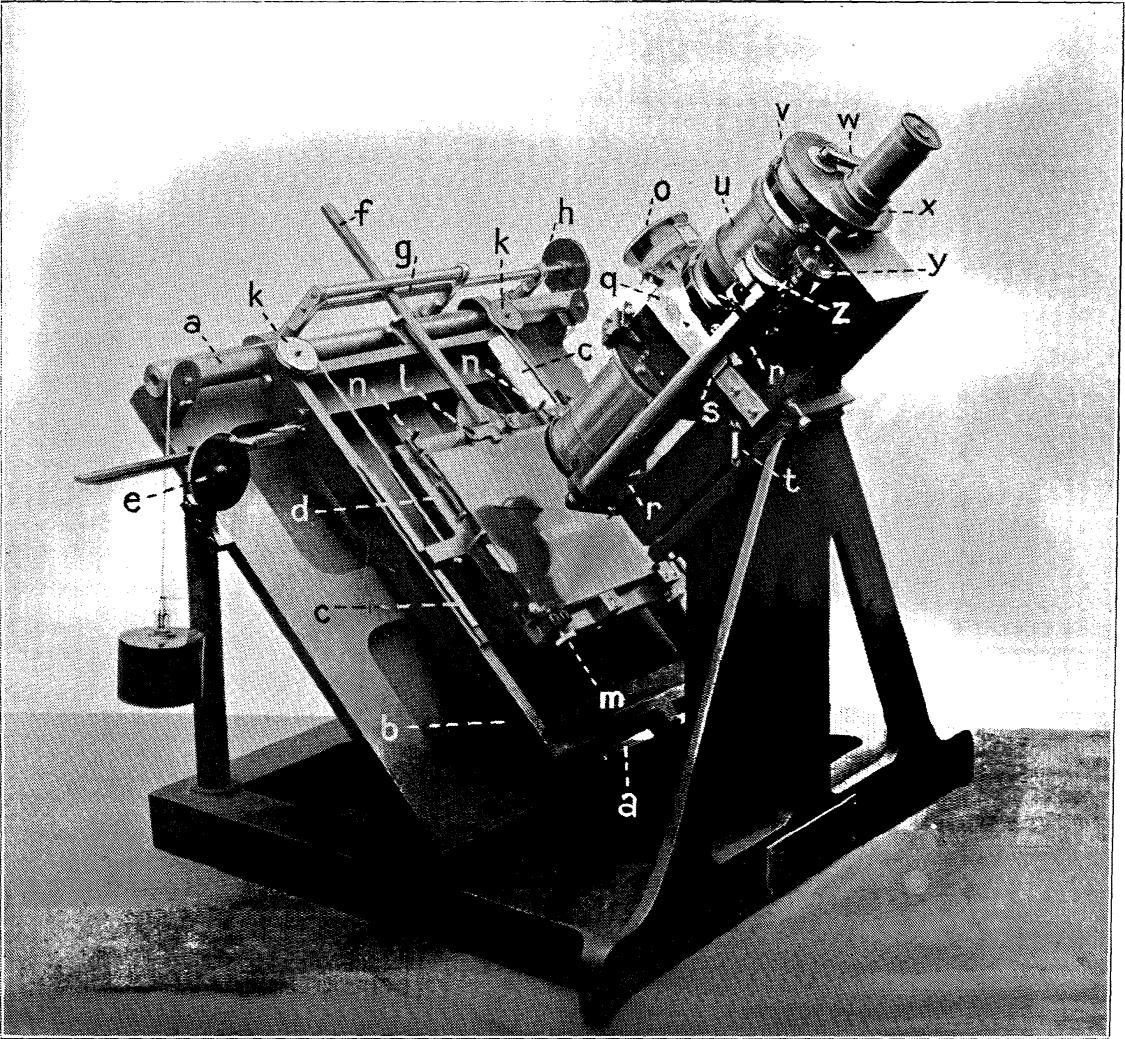
4. *The Construction of the Machine.*

The construction of the machine upon these lines was approved by the Director of the observatory, and the Cambridge Scientific Instrument Company, Limited, undertook the work. A draft of the features outlined above was sent, and plans embodying them were prepared by Mr. Horace Darwin. It is a pleasure to acknowledge the great debt which is due to him for the careful thought he has given to the design. Every detail of the machine was thoroughly discussed, and I think that all difficulties have been successfully overcome. Thanks are due to Mr. Newall for help in many ways; and several excellent ideas were borrowed from Sir David Gill's description of his new Repsold machine. Acknowledgment must also be made of the trouble which was taken by Messrs. Zeiss in arranging the optical parts, dividing the scale, and especially in designing for us a new form of reversing prism.

The accompanying photograph (Plate 12) shows the general appearance of the machine.

The base is a heavy iron casting resting on three feet, cast in one piece with the pillar which carries the microscope. Upon the base is erected a cast-iron frame which carries the main horizontal slides *a, a* of cylinder and plane form. On these rest a frame *b*, which carries the main vertical slides *c, c* of similar form, and on these slides rests the plate-carrier *d*.

The arrangements for giving motion to the plate-holder, and for counterpoising it on the vertical slide, demand a word of explanation. The frame *b* is moved along the horizontal slide by means of the rack and pinion *e*. The plate-holder carries a rack



THE CAMBRIDGE MACHINE FOR MEASURING CELESTIAL PHOTOGRAPHS

f which engages in a long pinion *g* parallel to the horizontal slide, and turned by the hand-wheel *h*. This gives a very nice motion in both coordinates at once. It is fairly quick, and at the same time delicate enough to do away with the necessity for a slow motion to the plate. There is a clamp on the vertical slide, which is useful when the plate is being run backwards and forwards on the horizontal slide to orientate it. But all the parts are so heavy that no clamp is required during measurement. To counterpoise the plate-holder on the vertical slide two cords are carried to pulleys *k, k*, at the top of the slide, and thence over pulleys at each side of the machine to two counterpoise weights. This is better than having one weight on a cord carried over the top. It reduces the weight to be carried on the horizontal slide, and does not interfere with the illumination from behind.

The plate rests upon three spring plungers, which brings its face up against three stops whose faces are worked to a plane parallel to the planes of the main slides, perpendicular to which the axis of the microscope is set. One of these stops, *l*, can be turned aside to permit the insertion of the plate. The lower edge of the plate rests at each end upon two pieces of long flat spring, which bear on the points of two adjusting screws *m* placed opposite the ends of the two extreme vertical lines of the *réseau*. Spring clips *n* on the top edge of the plate press it down against these springs, and the adjustment for orientation is easily made.

The microscope is in two parts, the body of each formed of turned steel tube, and resting in V bearings in a cradle fixed to the pillar. The four V's are accurately worked tangents to a cylinder. Fig. 2 shows the arrangement of optical parts and the position of the V's.

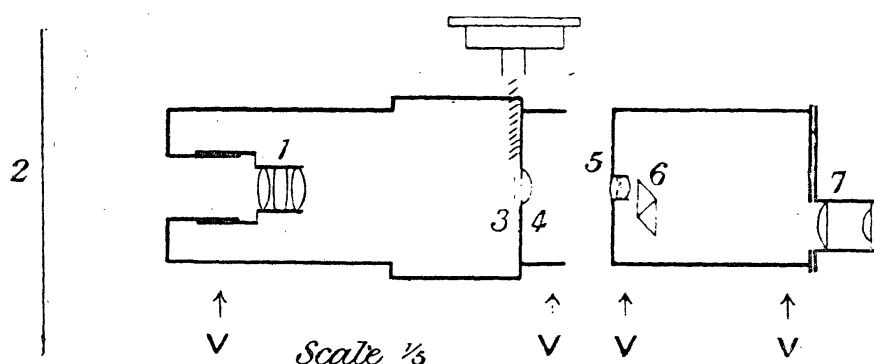


Fig. 2.

Objective I., at 1, is a Zeiss "symmetric anastigmat" combination of 60 mm. focal length. It projects an image of the plate 2 upon the divided glass scale at 3. The divisions, of course, are on the under side of the glass, so that the projection upon them is not distorted by the thickness of the glass. Immediately above the scale is a lens 4 termed by Messrs. Zeiss a

“collecting lens.” Its function is to converge the axes of the pencils of light diverging from all points of the glass scale, without altering appreciably the angle of divergence of the beams in each individual pencil. This is necessary in order that the whole of the pencils should be brought into the objective of the view microscope.

Objective II., at 5, is a Zeiss “symmetric aplanat” combination of 35 mm. focal length. Immediately above it is the reversing prism 6, which is of a new form, devised by Messrs. Zeiss. The usual place for a reversing prism is outside the eyelens. When it is placed there it cuts down the field of view very considerably, as is well known, and this was inadmissible. The new reversing prism has three reflecting surfaces, and it is an irregular solid very hard to figure. But its action may be

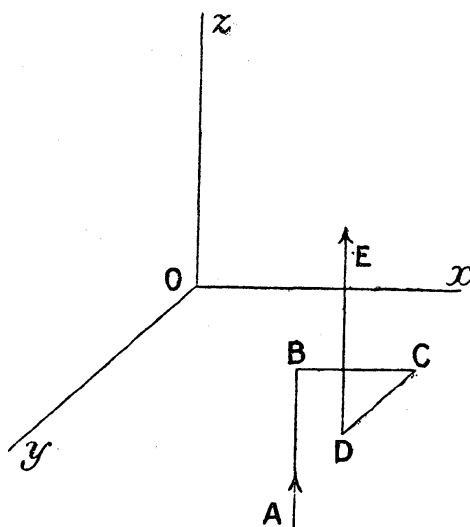


Fig. 3.

described thus : Let Ox , Oy , Oz be a system of rectangular axes. Let AB be the axis of the central pencil passing through objective II. It enters the prism at a face parallel to the plane xy . At B it is reflected parallel to the axis of x ; at C parallel to the axis of y ; and at D parallel to the axis of z ; so that it emerges along DE parallel to its original direction AB , through a face parallel to the face of entry. If these faces are large enough to take the diverging beam from the objective there is no loss of field of view ; and the definition of the microscope does not seem to be sensibly impaired by the action of the prism.

Since the optical axis is displaced parallel to itself, the eyepiece 7 must be mounted eccentrically on the end of the tube.

To return to the details of the mounting of these optical parts :

The objective 1 is mounted in a steel tube with four projecting studs, which bear outside a smaller steel tube fixed inside

the body of the microscope and turned concentric with it (see fig. 2). A spring keeps the two together, and a screw with divided head (seen reflected in the glass plate in the photograph) slides the objective tube along the other tube. This gives the motion of the objective required to fit the *réseau* square to the scale.

The double rectangular slide on which the divided scale is moved by the two micrometer screws is of a new pattern. It is geometrically desirable that the plane of the scale should coincide as nearly as possible with the plane of the slides, and this is difficult to manage if the slides are of the ordinary pattern, one crossing over the other.

The principle of the new slide is illustrated in fig. 4. The scale is held in a square block of steel, with two sides, AB, AC, ground accurately at right angles to one another, and to the face of the block. The points of the micrometer screws bear on these sides. Below the block is screwed a piece of steel forming two

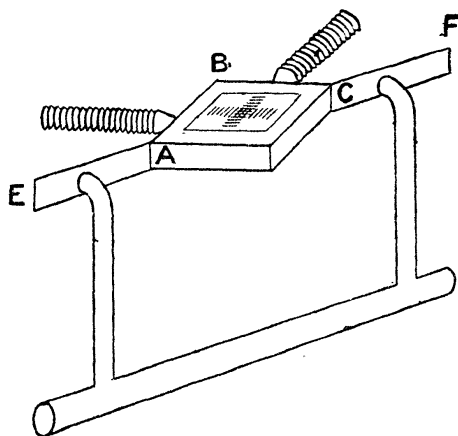


Fig. 4.

wings, AE, CF, with faces worked to a plane through the diagonal AC of the block, and perpendicular to its face. At some distance below is an axle working in V bearings parallel to AC; and from this axle spring two arms rigidly attached to it, which come up and bear against the faces of the wings. Spiral springs are wrapped round the axle, and the block is held against the points of the screws by the pressure of the arms upon the wings.

When either micrometer screw is turned the block is moved in a direction along the axis of the screw, and at the same time it is compelled to keep its diagonal AC always parallel to the axle from which the arms spring. The block has therefore motion in the plane ABC only in the two directions, mutually at right angles, of the axes of the micrometer screws.

Two long steel pins with sharp conical ends enter two shallow conical holes drilled in the under side of the block. The pins are driven upwards by spiral springs. This arrangement allows

ample lateral movement to the block, while it keeps its face ABC pressed against the worked inner surface of the top of the micrometer box, and it reduces friction of the slide to a minimum.

This new form of double micrometer slide is perhaps the most beautiful of the many pieces of ingenious geometrical design which Mr. Darwin has put into the machine. It is impossible to do justice to them in a brief sketch.

The micrometer screws *o* are of half-millimetre pitch. They were made by Messrs. Brown & Sharpe, of Philadelphia, U.S.A. The heads are of white celluloid divided into one hundred parts. Since the nut is fixed in the side of the micrometer box, and the motion transmitted by the point of the screw, the divided head has a considerable range of motion in the direction of its axis. The fiducial marks consist therefore of fine blackened silver wires stretched in frames and brought close down to the surfaces of the heads.

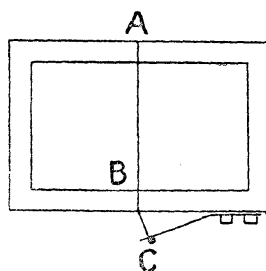


Fig. 5.

It is frequently required to mount fine wires in tension, and the ingenious method of fixing these wires used in this machine is worth describing. V-shaped cuts are made in the frame at A and B (fig. 5). C is a small piece of spring screwed to the side of the frame, and its end, which lies rather below the plane of AB, is pierced. A knot is tied in the wire; the wire is passed through the hole in C and laid in the cuts B and A; it is pulled up a little against the spring and fastened with a drop of solder at A. This arrangement keeps the wire always in tension.

The divided glass scale rests on three small projections in the edge of the square hole cut through the steel block. It is provided with an adjustment in orientation by which the scales may be set parallel to the axis of the micrometer screws. The complete glass scale will consist of two similar scales crossing at right angles. Up to the present Messrs. Zeiss have not been able to furnish an entirely satisfactory scale of this pattern, which seems to be difficult to make. The first crossed scale which they supplied consisted of two scales divided separately on thin glass and cemented face to face. This form was objectionable. It is essential that no thickness of glass shall be interposed between the scale and the objective, which projects on it the image of the plate. Otherwise there is introduced away from the centre of the field the distortion due to oblique passage through a parallel

plate, or perhaps to a plate not quite parallel. It did not seem worth while to incur the necessity of investigating and applying a troublesome small correction. The parts of the scale were therefore separated, and a single one is being used at present. Messrs. Zeiss hope to be able before long to supply the crossed scale ruled on one piece of glass.

The "collecting lens" in a steel cell is screwed into the top of the micrometer box *q*, so that it comes down very close to the scale.

This completes the description of the lower section of the microscope. The whole is free to turn in the lower pair of V's, *r*, *r* of the microscope cradle, through an angle of 90° . At the limits of its range of rotation it is brought up against stops *s*, adjusted so that the screws are parallel to the main slides, and spiral springs, *t*, can be hooked on to hold it in those two positions. It is possible therefore to measure the coordinates increasing with R.A. and Decl., or by turning through 90° with N.P.D. and R.A. The photograph shows the latter position. This power of rotation is of the greatest value in testing the centering of the somewhat complicated optical system.

The upper section of the microscope, with which the scale is viewed, consists of a low-power compound microscope, for the reasons outlined above. It is mounted in a steel tube *u* of the same diameter as the lower section of the microscope. Objective II. is screwed centrally into the lower end. Immediately above it is the reversing prism on a plate mounted on three screws by which the prism can be squared on to the optical axis. It should be remarked that this adjustment is not effective in the plane which contains the axis of the incident and emergent beams (the plane of fig. 2). A tilt of the prism in this plane produces no effect upon the direction of the emergent beam, which is controlled entirely by the angles at which the reflecting faces of the prism are worked.

The eyepiece plate *v* is double. A pin through the upper plate passes through an oval slot in the lower, and thence into a large flat washer. An arm *w* is attached to a nut on the upper end of this pin, and by giving it a slight turn the washer is tightened and the two plates clamped together. This makes a convenient slipping piece. If the eyepiece is held between the thumb and forefinger, the arm can be turned with the little finger and the nut loosened, the eyepiece slipped to the required position, and the whole clamped again.

There are three eyepieces which give powers, 20, 40, and 60 on the microscope. They are mounted on flanges *x* which have screws cut away like a gun breech, so that they slip home, and a slight turn holds them tight. Measuring is done with the power 20; but if any photographic defect in the image is suspected a higher power can be slipped in already adjusted to focus, and the suspected image examined.

The upper tube can be completely rotated in its V's. A turn through 45° turns the field of view through 90° by virtue of the

κ κ

reversing prism. A weak spring catch holds the tube lightly in the eight positions in which the scale and *réseau* lines appear in the microscope horizontal and vertical.

There are three separate adjustments for focus. The milled head y slides the upper tube along its V's with respect to the lower, and brings the divided scale into focus in the eyepiece. The divided head z slides both tubes together along their V's, and brings the plate into focus on the scale. Finally, the divided head seen reflected in the plate gives an independent motion to objective I. and corrects any error of runs.

The two latter heads are divided on celluloid into one hundred parts: the first, because it is difficult to focus the plate on the scale with extreme accuracy at a single setting: a number of settings are made, and the head left at the mean reading; the second head was divided so that a given error of run could be corrected immediately without a series of trials.

This completes the description of the microscope.

The setting scales on the main slides (one of them seen through the plate in the photograph) are of white celluloid, divided to mm., and numbered in R-intervals. The numbers run both ways, one set in black, the other in red, so that a star can easily be found in either the direct or reversed position of the plate. The indexes are adjustable.

Some care has been taken to shield the measurer from all unnecessary light. The instrument stands on a table covered with black cloth. A black curtain hangs immediately behind the machine, with only a small hole in it to admit light to a reflector beneath the plate. Another black curtain hangs to the left. The greater part of the machine is painted dead black. These precautions make it possible to keep open the eye which is not in use; no light falls on it to dazzle it, and there is nothing bright in sight to attract its attention. The comfort and absence of strain on the eyes are very marked.

The essential parts of the machine are made of steel or cast iron, very massive. The machine is little sensitive to temperature, and its rigidity is remarkable.

5. *Rapidity and Accuracy of Measurement.*

It is as yet a little early to estimate definitely the rapidity with which measures can be made with this machine. The plates which have so far been measured with it have been measured in different ways for experimental purposes, and no long series of plates has as yet been treated in a systematic way. At present I have adopted the plan of making one pointing on each of two R-lines and two pointings on the star image for each coordinate. Working in this way, and recording one's own measures, it will, I think, be easily possible to measure twenty-five stars per hour in both coordinates.* This is of course much

* [Note added later.] Further practice has increased the rapidity to thirty stars per hour.

less than Sir David Gill's record, eighty stars per hour, including diameters, though it must not be overlooked that it takes two observers to do this, one recording for the other. When allowance is made for this the Cape machine is probably getting on for twice as fast, and it will be well to attempt to show that increased accuracy compensates in the Cambridge machine for diminished rapidity. On the other hand, the Cambridge machine is a great deal faster than those used at the Paris and other French observatories, where they measure from twelve to fifteen stars per hour.

It may be mentioned as a matter tending to rapidity that there is no need to record the measures in all the detail given in the example (Section 2). For instance, the scale and screw readings may be added together before the figures are set down ; there is no need to take note for the second R-line of more than the last two figures of the micrometer reading ; and even these need not be set down—they may be compared with the reading on the first, and the run set down directly. And since the mean run is easily reduced to nearly zero, the run of individual squares will rarely amount to ten units in the last place, and the necessary correction may be determined by inspection (see the example later in the discussion of the Cape machine).

The proportions of screws and scales were designed on the supposition that $0^{\text{mm}}\cdot0005$ is just less than the smallest amount of shift between two well-defined marks which can be appreciated by the eye with a microscope magnifying twenty diameters. This should make it just not possible to set continuously on a well-defined point to the same estimated figure on the micrometer head, but the range of variation should not be more than a few units in the estimated place. Some experience in determining division errors has shown that this estimate is certainly not too high. The probable error of a single setting on a perfectly defined mark is not so high as $\pm 0^{\text{R}}\cdot0001$ ($=0''\cdot017$ on the plate) when the observer is in good form. This is high testimony to the stability and workmanship of the machine ; but it is of course no criterion as to the accuracy which may be expected in measures of star plates. Professor Kapteyn has given in *Publications of the Astronomical Laboratory of Groningen* (pt. i. p. 81) some interesting figures to show that the real accuracy in the measured position of a star image is much less than the accuracy estimated from the divergence of successive pointings on it, or even from the agreement of measures made in reversed positions of the plate ; that is to say, the position of a star image has a real error of its own, independent of the errors of measurement, due to such causes as irregularities in the size and arrangement of the silver particles. The true P.E. of a measured position on the plate, deduced by comparison with the results of other plates, is therefore not a fair test of the capabilities of the machine ; it depends more upon the defining power of the telescope, atmospheric disturbances, &c., as well as the quality of the plates.

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The accuracy of the measuring machine is sufficient for its purpose if the errors in pointings due to the machine are much less than those due to irregularities in the star image.

Judged by this standard the Cambridge machine is amply accurate. A first determination of the P.E. of a complete measure (one setting on each R-line and two on the star in each position of the plate) derived from a direct reduction of one plate to another by means of eleven common stars, is $\pm 0^{\text{R}}.0004$, $= 0''.07$; whereas the P.E. of a *single setting* on a well-defined mark is less than $\pm 0^{\text{R}}.0001$. It is to be concluded that no gain in accuracy would result from making the micrometer screws finer.

6. Errors of Screws and Scales.

a. Progressive Error of Screw, and Ratio of Screw to Scale.—A well-defined speck was chosen on the plate, and every tenth division of one of the scales from 150 to 50 was set upon it in turn by the screw, the plate remaining fixed. The values of successive distances of ten-scale spaces in terms of the screw revolution were found to be 1^r.003, 3, 2, 3, 2, 2, 2, 1, 2, 1; the small variations in these distances include of course division errors of scale as well as progressive errors of screw. There is evidently no appreciable progressive error in the screw over ten revolutions unless it should chance to be exactly compensated by a similar error in the scale; and this is shown later not to be the case. The pitch of the screw differs from that of the scale by about two-tenths per cent., which is negligible, since only fractions of a screw revolution are used.

b. Division Errors of the Scale.—These were partially examined by the method used by Sir David Gill in his investigation of the heliometer scales (*Monthly Notices*, xlix. p. 110). An etched scale by Zeiss was mounted on a plate and used as the auxiliary scale. The space between 150 and 50 was taken as correct, and the error of every tenth intermediate space found. The means of two determinations, which agreed well, gave for the corrections $-12, +4, +9, +3, +3, +17, +13, +32, +8$, all multiplied by 10^{-5} R-intervals. Only one of these is at all large, and that is due to an obviously bad cut of one division, which would be avoided in use.

A similar examination of the error of every alternate space from 110 to 90, assuming the errors for these spaces found above, gave for the intermediate space errors $+9, +12, +14, +3, -3, -4, -8, +13, +6 \times 10^{-5}$ R-intervals.

It is evident that the division errors of the scale examined rarely amount to more than one in the last place of decimals used ($.0001 \text{ R} = .0005 \text{ mm.}$). It seems safe to treat these errors as accidental.

c. Periodic Errors of Screws.—An examination was made of the screw used with the single scale at present in use. No periodic term could be detected having a coefficient so large as a

single thousandth of a screw revolution. If there are periodic errors they are so small that they could not be determined without using a reading microscope to read the divisions of the head to a greater accuracy than estimation of tenths gives. The screw may therefore be pronounced perfect.

7. *Comparison between the Cambridge Machine and Sir David Gill's Repsold Machine.*

It is not fair to compare directly the performances of the Cape and Cambridge machines, because the former was designed for the measurement of the astrographic catalogue plates, and something is admittedly sacrificed to speed. On the other hand the Cambridge machine was made for measuring plates for parallax determinations, and the highest attainable accuracy is the first consideration. It is, however, admissible to suggest that it seems probable that the principle of the Cambridge machine could be adapted to the Cape work, with a gain in accuracy, and no loss of speed.

Professor Turner has brought forward several objections to the Cape machine (*Monthly Notices*, lix. p. 135, 1899 Jan.). The weightiest is the conclusion that on Sir David Gill's own showing (*Monthly Notices*, lix. p. 73, 1898 Dec.) the micrometer screws of his machine should be already worn out. It seems to me that this estimate is exaggerated. It is based upon a comparison between the microscopes of the meridian circle and the measuring machine. In the former, if a complete determination of run is made for each reading, the screw must be turned through a range of 5 rev. backwards and forwards each time. In the latter the screw is turned only from star to star within the R-square, and is not set upon the R-lines themselves. The criticism appears, however, to apply precisely to the use of the screws in the Paris and Potsdam machines. In any of the cases the wear of the screw soon spoils the accuracy of the machine.

A second objection which Professor Turner brought forward is the partial neglect of error of run. Sir David Gill gives reasons for believing that its effect is small. Reference to his paper, p. 67, shows that he adjusts his square of webs to fit the mean R-square on the plate, and that if any individual R-square does not fit the webs he makes a symmetrical pointing on it; the position of the star image should thus be referred to the middle of the square, and the effect of run halved. The efficacy of this procedure seems to me somewhat doubtful. In placing the R-square on the square of webs attention must be concentrated on one line or the other, and the accuracy of setting on the one is considerably greater than the power of perceiving afterwards a want of symmetry in the other—at least, it seems so to me. It is probable that on this account, when the runs are small, the setting is made on one line, and the symmetry of the other is regarded as sufficient when it is really not so. The

error of run would then come in to its full amount. This remark does not of course apply to large errors of run, which are readily enough perceived, but to the continual small changes, due to distortion of film, errors of R-lines, and want of flatness of plate, which are evident to measurement, but scarcely apparent to inspection.

I venture, with some diffidence, to suggest that the principle of the combined scale and screw might be used in the Cape type of machine with very little, if any, loss of speed and some gain in accuracy. At the Cape Observatory the measurers work in pairs, one acting as clerk to the other. Suppose the measures made exactly as I have described in paragraph 2, and let us take the figures used in that example. The observer makes the setting on the first R-line and calls out 52,349; the recorder enters this at once as 5549. The observer now sets on the other R-line with results 152,341; the recorder need only do the simple subtraction of the last figures in his head, $49 - 41 = +8$, and record "run +8." Finally the observer sets on the star with readings 118,502. The recorder enters these as 12302, and while the observer is finding the next star performs the subtraction $12302 - 5549 = 6753$, adds 5 for run, and writes down the corrected result 6758. The observer has to make one more setting per star, three instead of two; but on the other hand he saves the time spent in giving several turns to the screw. The recorder has more work to do, but I think he could keep pace with the observer after a little practice. The result should be more accurate, since runs would be rigidly allowed for; the screw would not wear out nearly so quickly, and there would be no anxiety about permanence of somewhat complicated adjustments.

The rapidity of measurement which might, I think, be attained with the Cambridge machine used in this way depends of course on the condition that two people are available for the work. It is rather a special case which was not contemplated in the work for which the machine was designed. But the very satisfactory results which the combination of screw and scale has given at Cambridge seem to be some excuse for entering rather at length into the question whether their use might not be the means of avoiding one or two of the objections which have been raised to the methods adopted at the Cape Observatory, and more recently at Sydney and Melbourne, for the measurement of the Astrographic Catalogue plates.

I am indebted to the Cambridge Scientific Instrument Co. for the negative from which the plate illustrating this paper was prepared.

Cambridge Observatory : 1901 May 3.

Note on the Geometry of the Siderostat. By H. C. Plummer, M.A.

1. In the following note I have collected some of the principal geometrical propositions which relate to the siderostat and the motions connected with that instrument. There is little that can be considered essentially new in this account, but at the same time it may not be altogether useless to attempt to put the theory in a simple and clear light. Hitherto the siderostat has engaged the attention of the physicist rather than of the astronomer, and it is in France especially that its development has been promoted, while in this country it has been comparatively neglected. For these reasons there is perhaps some excuse for considering the matter in this place.

2. The principal propositions to which reference is made, and which are nearly all quite well known, are these :

1. The normal to the mirror describes a cone of the second order.

2. The circular sections of this cone are (*a*) perpendicular to the axis of the Earth, and (*b*) perpendicular to the ray reflected in a fixed direction.

3. The section (*a*) is described with uniform motion.

4. The section (*b*) is described with motion derived by stereographic projection from uniform motion.

5. The intersection of the mirror with the plane of reflexion describes a cone of the second order.

6. The circular sections of this cone are (*c*) perpendicular to the axis of the Earth, and (*d*) perpendicular to the ray reflected in a fixed direction.

7. The section (*c*) is described with uniform motion.

8. The rotation of the field is precisely the same as the motion with which the section (*b*) is described.

3. In the figure (fig. 1) which illustrates this note all directions are referred to a sphere of which the centre is C, the fixed centre of the siderostat mirror. The instrument is so adjusted that the star S, whose N.P.D. is δ , is reflected in the fixed direction CA. Hence the normal to the mirror CN bisects the arc SA. The pole is P, and is reflected to Q, so that the arc PQ is also bisected at N. It is at once obvious that AQ is equal to PS, and that Q therefore describes a circle round A of angular radius δ . The lines ACA', PCP' are diameters of the sphere. The intersection of the mirror with the plane of reflexion is represented by CM. This line is perpendicular to CN, and lies in the plane ASA', which is clearly the plane of reflexion.

4. All the geometrical properties enumerated above become evident when the figure is examined. The lines A'S and CN are parallel for $SA'A = \frac{1}{2} SCA = NCA$. Similarly P'Q and CN are parallel for $QP'P = \frac{1}{2} QCP = NCP$. Hence the three